

# AN APPROXIMATE APPROACH OF HEAT TRANSFER ACCOMPANIED BY PHASE TRANSITION

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**ABSTRACT** LOW TEMPERATURE HEAT (SOLAR ENERGY, GEOTHERMAL ENERGY, INDUSTRIAL AND DOMESTIC WASTE HEAT) IS WIDELY AVAILABLE FOR MANY APPLICATIONS. ENERGY STORAGE OF THAT HEAT IS NOW OF GREAT IMPORTANCE. PHASE CHANGE MATERIAL (PCM) IS PARTICULARLY ATTRACTIVE DUE TO ITS ABILITY TO PROVIDE A HIGH-ENERGY STORAGE DENSITY AND ITS CHARACTERISTICS TO STORE HEAT AT A CONSTANT TEMPERATURE. IN THE PRESENT STUDY, THE PHASE CHANGE PROBLEM IS ANALYSED IN TYPE OF SHELL-AND-TUBE HEAT STORAGE SYSTEM WITH CLOSED TUBES (CAPSULES) FILLED WITH PCM AND THE HEAT TRANSFER FLUID (HTF) FLOWING IN THE SHELL SPACE. A TWO DIMENSIONAL MATHEMATICAL MODEL IN CYLINDRICAL COORDINATES WITH A SOURCE MEMBER IS USED TO DESCRIBE THE PHYSICAL PROCESSES. A COMPUTER PROGRAM BASED ON THE THEORETICAL MODEL IS DEVELOPED TO PREDICT THE TRANSIENT BEHAVIOUR OF HEAT TRANSFER BETWEEN HTF AND PCM. A SERIES OF NUMERICAL EXPERIMENTS IS UNDERTAKEN TO ASSESS THE EFFECT OF VARIOUS THERMAL AND GEOMETRIC PARAMETERS ON THE TRANSFER PROCESS. THE COMPUTATIONAL MODEL IS VALIDATED WITH EXPERIMENTAL DATA.

## I. Introduction

Low temperature heat (solar energy, geothermal energy, industrial and domestic waste heat) is widely available for many applications. Energy storage of that heat is now of great importance because the key to the effective and widespread use of low temperature heat is its adaptation to the energy requirements. From this point of view, storage tanks based on the phase change principle are one interesting alternative. Phase change material (PCM) is particularly attractive due to its ability to provide a high-energy storage density and its characteristics to store heat at a constant temperature corresponding to the phase transition temperature of the heat storage material. That is why the problem of the heat transfer accompanied by phase transition is of mine interest to the practice.

Heat storage system with the PCM and shell-and-tube type is analysed by Lacroix [6]. Several authors, using mathematical models of different complexity, studied this type of latent heat storage unit. Heat-of-fusion storage materials for low temperature storage in the temperature range 0-120°C are reviewed by Abhat [5] and Reiter and Rota [4]. Hamdan and Elweer [3] have investigated a melting process of a solid phase.

In the present study, the phase change problem of the PCM is analysed in another type of heat storage system. We use closed tubes (capsules) filled with PCM and the heat transfer fluid (HTF) flowing in the shell space of the heat exchanger (Fig.1). The heat process is analysed in terms of both radial and axial direction and is linked to the convective heat transfer from the HTF.

As it is known [2,3] the transient heat transfer in PCM can be described by the well-known heat conduction equation applied for various phase conditions and the energy balance on the interface given by the Stephan's equation. The latter determines the rate of moving of the interface and in this manner the space areas where the heat conduction equations have corresponding coefficients.

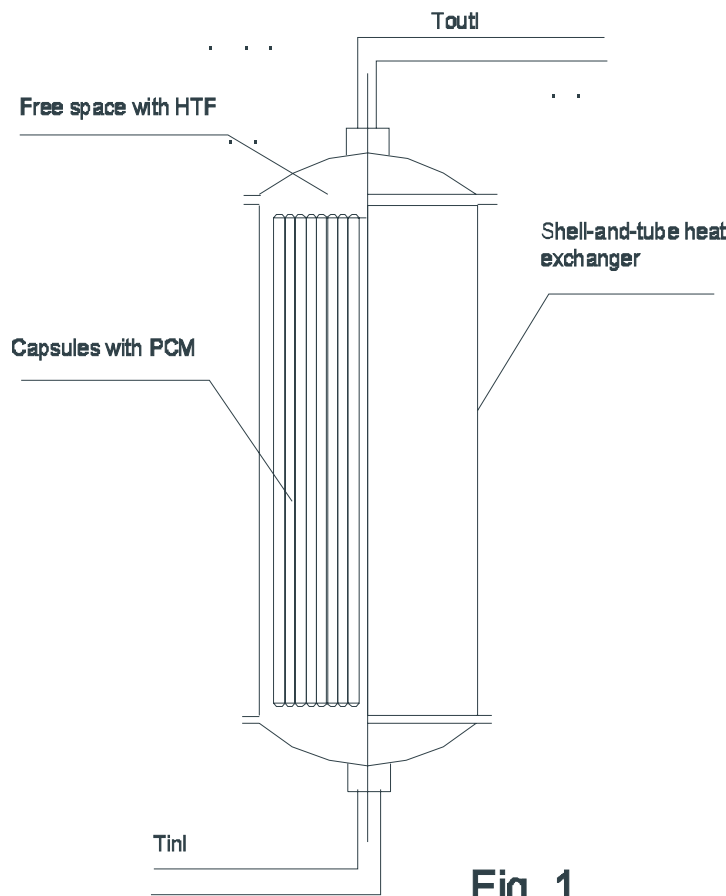


Fig. 1

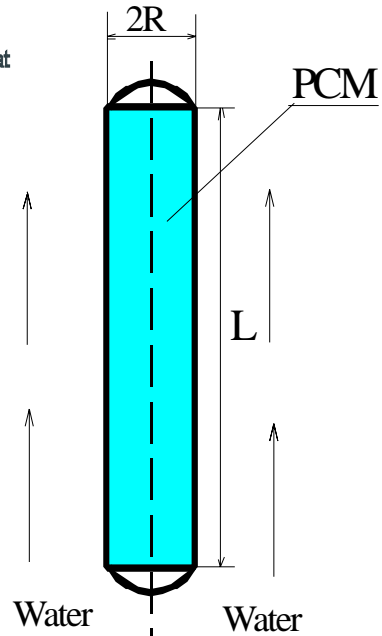


Fig. 2

The application of this approach is related to the use of two space areas of solutions and leads to considerable difficulties in the algorithm and programming in the cases of two and three-dimensional problems. Another method for the solving of this kind of problems is the use of equivalent characteristics of the substances [3,4]. This approach is based generally on the evaluation of the equivalent specific heat capacity of the substance in the phase change area summarizing in this manner the heat capacity effect and the heat for the phase transition. This treatment has the advance to obtain a homogeneous mathematical problem having a common space solution area. It should be mentioned that the equivalent specific heat capacity is strongly temperature depending and so a supplementary dependence between space and time increment can arise during the discretization of the equations.

The aim of this study is therefore to propose a new approach to the problem, using a source member in the heat conduction equation that takes in account the effect of phase transition.

## II. Mathematical modelling

The heat storage considered in the present study is shown in Fig.1. The PCM fills tubes (capsules) and HTF flows in shell space. The tubes are staggered so that around each there is a boundary (dotted line). In the real system were used gofer tubes and HTF can reach all outer surfaces of capsules. In this case physical system to be analysed is represented by the simpler geometry shown in Fig. 1. A shell-and-tube heat exchanger is imagined to be a cylindrical capsule with radius  $R$ , length  $L$ , which is contacted with HTF flowing vertically.

The two dimensional heat conduction equation with a source member can be written in

cylindrical coordinates as follows

$$\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial r} (a(\tau, r, z) \frac{\partial T}{\partial r}) + \frac{1}{r} a(\tau, r, z) \frac{\partial T}{\partial r} + \frac{\partial}{\partial z} (a(\tau, r, z) \frac{\partial T}{\partial z}) + f(\tau, r, z) \quad (1)$$

Where  $\tau$  is time variable,

$r$  and  $z$  are space coordinates (cylinder radius and height)

$a(\tau, r, z) = \lambda \rho c$  - heat conductivity parameter;

$f(\tau, r, z)$  - heat source member, which correspond to the phase change heat energy on the phase change surface.

For crystalline substances the phase change occurs at constant temperature  $T_m$  and function  $f$  must compensate the right side members of equation (1):

$$f(\tau, r, z) = -[\frac{\partial}{\partial r} (a(\tau, r, z) \frac{\partial T}{\partial r}) + \frac{1}{r} a(\tau, r, z) \frac{\partial T}{\partial r} + \frac{\partial}{\partial z} (a(\tau, r, z) \frac{\partial T}{\partial z})] \quad (2)$$

for  $\tau, r, z \in s(\tau, r, z)$  and  $f(\tau, r, z) = 0$  for  $\tau, r, z \notin s(\tau, r, z)$

where  $s(\tau, r, z)$  is a phase change surface, which separate solid and liquid phases of storage material.

Substituting /2/ in /1/ the later can be rewritten for the interface:

$$\frac{\partial T}{\partial \tau} = 0 \quad \text{or} \quad T = T_m$$

For  $r = 0$  (the axe of the cylinder) the assumption of symmetry of the solution leads to (for  $r$  variable only):

$$\frac{\partial T}{\partial \tau} = 2 \frac{\partial}{\partial r} (a(\tau, r, z) \frac{\partial T}{\partial r}) + f(\tau, r, z) \quad (3)$$

where

$$f(\tau, r, z) = -[\frac{\partial}{\partial r} (a(\tau, r, z) \frac{\partial T}{\partial r})]$$

for  $\tau, r, z \in s(\tau, r, z)$  and  $f(\tau, r, z) = 0$  for  $\tau, r, z \notin s(\tau, r, z)$

From the physical conditions for process in a PCM storage device the applicable boundary conditions can be drawn:

$$\text{For } r = R \quad \lambda \frac{\partial T(\tau, r, z)}{\partial r} = K(T_f(\tau, z) - T(\tau, r, z)) \quad (4)$$

$$\lambda \frac{\partial T a(\tau, r, z)}{\partial z} = 0 \quad (5)$$

for  $z = 0$  and  $z = L$

As far as the physical significance of the above boundary conditions is concerned, we may note that the boundary condition (4) determines the heat exchange between the surface of the storage device having a temperature  $T(\tau, R, z)$  and the flowing fluid with an inlet temperature  $T_f(\tau, 0) = T_{f0}$ . The overall heat transfer coefficient  $K$  takes in account both heat transfer by convection and heat conduction through the wall of the capsule containing the phase change material. According to (5) the

heat transfer through both faces of the capsule ( $z = 0$  and  $z = L$ ) can be neglected, which corresponds to thermal insulated surfaces of the capsules.

The temperature change in the fluid along the height of the cylinder can be expressed using the equation of the momentary energy balance of the fluid:

$$K\pi R^2(T(\tau, r, z) - T_f(\tau, z)) = C_f G \frac{\partial T(\tau, z)}{\partial z} \quad (6)$$

where  $C_f$  is specific heat capacitance of the fluid and  $G$  is a fluid flow rate.

### III Numerical approximation of the equations

The system (1) - (6) can be solved by the finite difference method. For these purposes, equation (1) takes the form:

$$\frac{\partial T}{\partial \tau} = L_r T + L_z T + f \quad (7)$$

where

$$L_r = \frac{\partial}{\partial r} \left( a \frac{\partial}{\partial r} \right) + \frac{1}{r} a \frac{\partial}{\partial r} \quad L_z = \frac{\partial}{\partial z} \left( a \frac{\partial}{\partial z} \right)$$

are differential operators.

According to the difference set of the solution area (Fig.3) the general finite difference approximation of these differential operators gives:

$$Y_r = \frac{2\sigma}{r_i(\Delta r_i + \Delta r_{i+1})} \left[ a_{i+1} \left( r_i + \frac{\Delta r_{i+1}}{2} \right) \frac{T_{i+1}^{n+1} - T_i^{n+1}}{\Delta r_{i+1}} - a_i \left( r_i - \frac{\Delta r_i}{2} \right) \frac{T_i^{n+1} - T_{i-1}^{n+1}}{\Delta r_{i+1}} \right] \quad (8)$$

$$+ \frac{2(1-\sigma)}{r_i(\Delta r_i + \Delta r_{i+1})} \left[ a_{i+1} \left( r_i + \frac{\Delta r_{i+1}}{2} \right) \frac{T_{i+1}^n - T_i^n}{\Delta r_{i+1}} - a_i \left( r_i - \frac{\Delta r_i}{2} \right) \frac{T_i^n - T_{i-1}^n}{\Delta r_{i+1}} \right]$$

$$Y_z = \frac{\sigma}{\Delta z} a_{j+1} \frac{T_{j+1}^{n+1} - T_j^{n+1}}{\Delta z} - a_j \frac{T_j^{n+1} - T_{j-1}^{n+1}}{\Delta z} + \frac{1-\sigma}{\Delta z} a_{j+1} \frac{T_{j+1}^n - T_j^n}{\Delta z} - a_j \frac{T_j^n - T_{j-1}^n}{\Delta z} \quad (9)$$

where  $\sigma$  is weight coefficient of the finite difference scheme. For  $\sigma = 0.5$  a symmetric finite difference approximation (the so-called Crank-Nicolson formulation) may be obtained [2]. It is known that the latter give the higher accuracy of approximation for the differential operators. The subscripts  $i, j, n$  were corresponding to the independent variables  $r, z$  and  $t$ . Both (8) and (9) were simplified by omitting the subscript  $j$  for the temperature in (8) and the subscript  $i$  in (9).

The general approximation takes the form:

$$\frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta \tau} = Y_z T + Y_r T + f^{\wedge} \quad (10)$$

where  $f^{\wedge}$  is the approximated analogue of the source function represented by the operators  $Y_r$  and  $Y_z$  following the eqn. (2). Equation (3) can be approximated as follows:

$$\frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta \tau} = \frac{4}{\Delta r} a_1 [\sigma(T_{2,j}^{n+1} - T_{i,j}^{n+1}) + (1 - \sigma)(T_{2,j}^n - T_{i,j}^n)] \quad (11)$$

Finite difference approximations of the boundary conditions are from essential importance, because of achieving an accuracy of approximation scheme corresponding to the accuracy of approximation of eqn. (8) and (9). Use of Taylor's series and main equation (1), the differential terms in eqn. (4) and (5) can be presented as follows:

$$\frac{\partial T(\tau, r, z)}{\partial r} = \frac{1}{1 - \frac{\Delta r}{2R}} \left[ \frac{T(\tau, R - \Delta r, z) - T(\tau, R, z)}{\Delta r} - \frac{\Delta r}{2 a(\tau, R, z)} \frac{\partial T(\tau, R, z)}{\partial \tau} \right]$$

$$\frac{\partial T(\tau, r, z)}{\partial z} = \frac{T(\tau, r, z1 - \Delta z) - T(\tau, r, z)}{\Delta z} - \frac{\Delta z}{2 a(\tau, r, z1)} \frac{\partial T(\tau, r, z1)}{\partial \tau}$$

(12)

The approximation of the boundary condition can be obtained by introducing above equations in (4) and (5) and replacing terms by their finite difference approximations. Here is presented approximation form of eqn. (4):

$$\sigma K (T_{f,j}^{n+1} - T_{i,j}^{n+1}) + (1 - \sigma) K (T_{f,j}^n - T_{i,j}^n) = X1 \sigma (T_{i,j}^{n+1} - T_{i-1,j}^{n+1}) + X1 (1 - \sigma) (T_{i,j}^n - T_{i-1,j}^n) + X2 (T_{i,j}^{n+1} - T_{i,j}^n) \quad (13)$$

where

$$X1 = \frac{\lambda}{(1 - \frac{\Delta r}{2R}) \Delta r} \quad ; \quad X2 = \frac{\lambda \Delta r}{2 (1 - \frac{\Delta r}{2R}) \Delta r} a_{i,j} \Delta \tau$$

and  $\lambda$  is heat conductivity coefficient of the PCM.

Despite the fact that difference approximation (10) satisfies the stability requirements when the  $\sigma = 0.5$ , some particular conditions have to be considered. It is necessary that increments  $\Delta r$  and  $\Delta t$  have to be determined according to:

$$\sigma \leq \frac{K - X2}{K + X1}$$

The algebraic system received after the finite differential approximation can be solved using fractional increment method [2,3]. For these purposes the two-dimensional problem may be handled by successively solving of two one-dimension cases. Mathematically it means that eqn. (10) is replaced by:

$$\frac{U_{i,j}^{n+1} - U_{i,j}^n}{\Delta \tau} = Y_r U + f^r \quad \text{for } U_{i,j}^{n+1} = T_{i,j}^n \quad (14)$$

and

$$\frac{V_{i,j}^{n+1} - V_{i,j}^n}{\Delta \tau} = Y_r V + f^{\wedge}_z \quad \text{for } V_{i,j}^n = U_{i,j}^{n+1}; \quad T_{i,j}^{n+1} = V_{i,j}^{n+1} \quad (15)$$

For known temperature  $T_{i,j}^n$  the first step consists in solving the system (14) to obtain the intermediate values of the temperature. In the second step the system (15) is solved for  $V_{i,j}^n = U_{i,j}^{n+1}$  and the desired temperature values  $T_{i,j}^{n+1} = V_{i,j}^{n+1}$  are obtained.

#### IV Position of the interface

The radial motion of the interface between solid and liquid phase of the storage material is here accounted by the source function in eqn.(1). The cross cut of the cylinder with one part of the difference set is given on Fig.1. It is important to note that the solving of the system of algebraic equations includes constant increments on  $z$  and  $t$  but not on  $r$  variable. This increment can be constant on all solution net with one exception - the area near the interface. In this area variable increment on  $r$  are applied. For these purposes the net line near the phase change surface is excluded from the consideration and is replaced by the latter. This is presented on Fig.2, where the surface having a radius  $r_i$  is replaced by the phase change surface having a radius  $r_s$ .

The new position of the phase change surface can be determined after the evaluation of the temperature  $T_{i,j+1}^n$  and source member  $f^{\wedge}$ :

$$r_s = \sqrt{\frac{\pi r_s \rho \Delta z - f^{\wedge} / Q_m}{\pi \rho \Delta z}}$$

where  $\rho$  is density of the phase change material.

**V Result and discussions** Results presented by Abhat [5] show that paraffin is very suitable material for low temperature applications, wherein temperature intervals of the store are generally limited to 10-15 K about the melting point. The melting temperature varies in wide range (30 - 65 K) depending on comprises chemical compounds. Paraffin has large heat of fusion - in range 150 - 190 kJ/kg. That is why the paraffin is chosen for PCM in presented study.

Using the model described above, a computer program was created to predict the transient behaviour of water to PCM exchanger unit. A series of numerical experiments was undertaken to assess the effect of various thermal and geometric parameters on the processes.

The computational model was validated with experimental data. An experimental storage unit was constructed and some experiments were performed. The scheme of the storage unit is shown in fig. 3. The main parameters of the storage are: number of tubes - 30; diameter of tubes - 0.006 m;

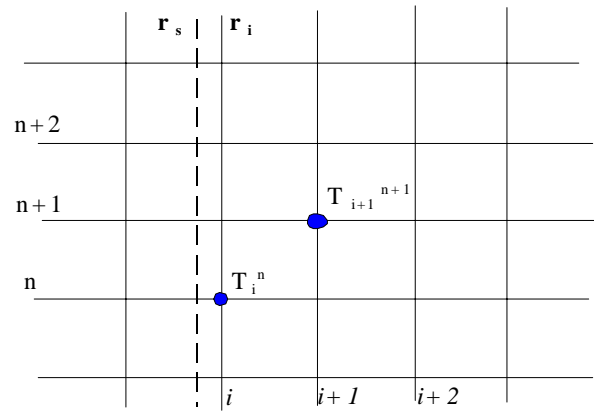


Fig. 3

length of tubes - 1m. Water to PCM heat exchanger is imagined, as it mentioned, to consist of cylindrical capsules stacked to form vertically heated surfaces. The main properties of the PCM were melt temperature -  $T_m = 49 - 50 \text{ }^\circ\text{C}$ , latent heat 140 - 156 kJ/kg. Temperature of HTF was 60 C.

In the next graphics are presented some of theoretical results received by the computer program. Figure 4 shows the interface position in the different moments of the melting process. Interface position is calculated by above-mentioned technique.

After a thorough investigation, appropriate values and formulas for coefficients in mathematical equations and parameters of numerical approximations were chosen. A satisfactory coincidence between experimental and theoretical results was achieved. Since there are difficulties to equalize the initial conditions of the mathematical system with real parameters of accumulators, we assume that the correlation between the theoretical predications and measured data is good.

The goal of theoretical experiments was to select an appropriate construction and thermal characteristics of water to PCM exchanger unit. In order to do this, a comparison between water heat accumulator and PCM accumulators with different characteristics was made. Figure 5 shows the comparative results from calculations of three types of PCM accumulators and water accumulator with equal volume. Heat capacity depends on free space (water content).

Figure 6 shows the effects of temperature difference between HTF temperature and melting temperature of the PCM. This temperature difference determines the time of full melting (charge - discharge) of the PCM in accumulators and heat intensity of charging (discharging) process. The other base parameter of working fluid influencing the phase change process is flow rate. Figure 7 describes the effect of flow rate variation.

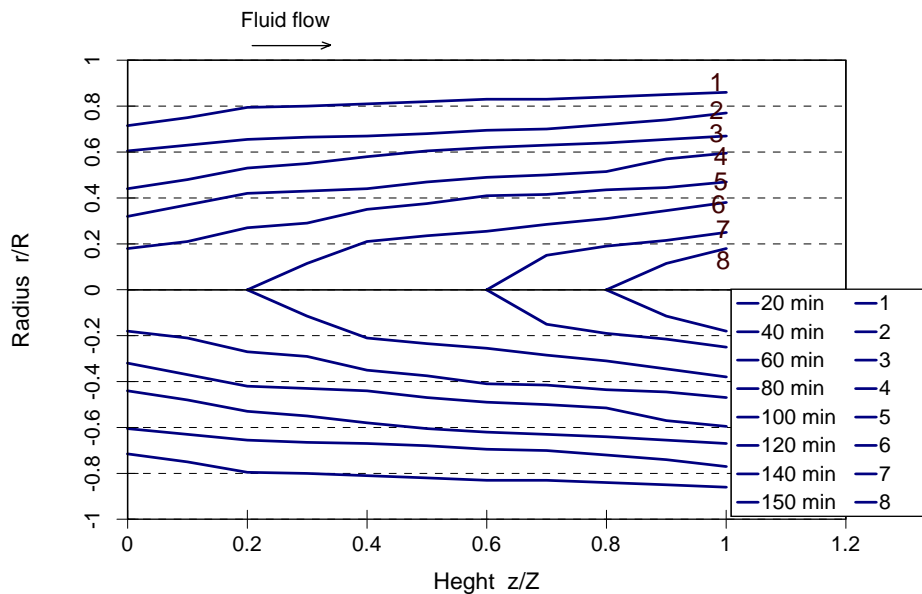


Fig. 4 Position of the interface

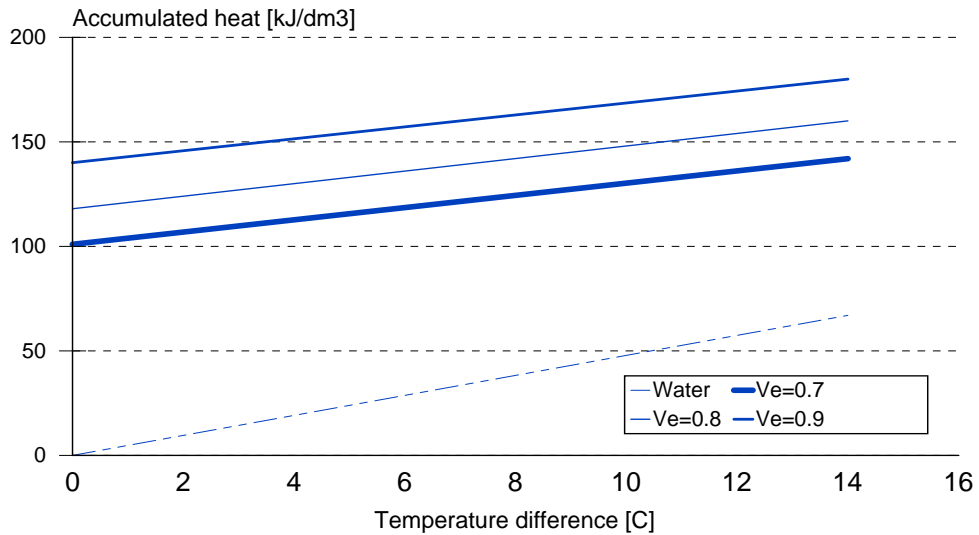


Fig. 5 Heat capacity of storage with different free space  $V_e$

### VI Conclusions

A theoretical model was developed to predict the transient behavior of shell-and-tube storage unit, with the PCM filling the tubes and the HTF circulating shell side. The phase change problem of the PCM is treated in both radial and axial directions using energy balance method and is strongly

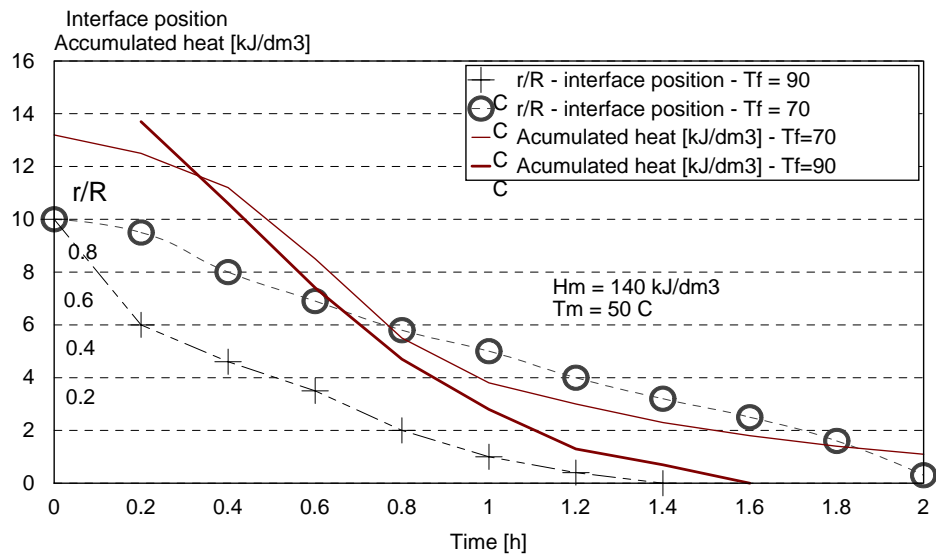


Fig.6 Parameters of melting in heat storage

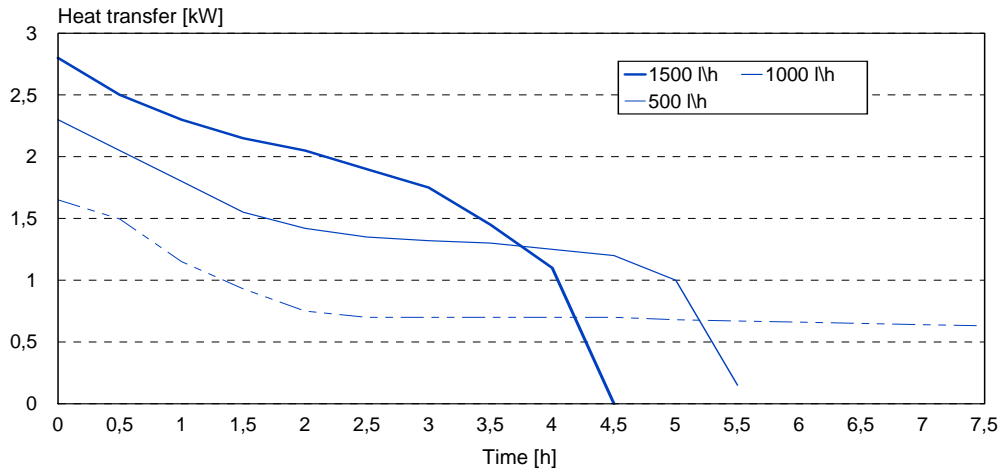


Fig.7 Heat transfer rate depending on HTF flow rate

coupled to the convective heat transfer from the HTF. A computer program for simulation calculations was developed, and the applicability of the presented model was verified with a large number of numerical experiments.

The present theoretical analysis may be used to predict melting rate in the water to PCM exchanger unit. As a result this analysis will be used in order to predict the amount of thermal energy stored within a storage system containing a phase change material.

The results from presented study would help the designers of heat systems (heating in buildings, hot water) to select optimal construction parameters of heat accumulator. We are expecting, that that model and computer program will allow us to expand the scope of designers calculation methods (SLR - method, U-U - method) for different constructions of heating and solar systems.

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